UDC 533.6

PERTURBATIONS IN A SUPERSONIC FLOW DUE TO DISCRETE OR CONTINUOUSLY DISTRIBUTED MASS AND HEAT SOURCES

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Inzhenero-Fizicheskii Zhurnal, Vol. 8, No. 5, pp. 684-686, 1965

The equations describing the flow of a perfect gas in the pressure of mass and heat sources due to combustion of fuel injected into the stream, without account for body forces and the energy lost in the displacement of gas with mass addition, have the form

$$\frac{d\ln\rho}{dt} + \operatorname{div}\vec{w} = \dot{m},$$

$$\frac{d\vec{w}}{dt} = -\frac{1}{\rho}\operatorname{grad} p,$$

$$\frac{d}{dt}\left(c_p T + A\frac{w^2}{2}\right) = A\frac{1}{\rho}\frac{\partial p}{\partial t} + \dot{q} + i_{\text{от}}\dot{m},$$

$$p = g\rho RT.$$

It is assumed that the regions of mass and heat supply coincide, and that the added mass velocity projected in the direction of motion is small compared with the free-stream velocity.

In the case of small perturbations, assuming that the motion is steady and that the flow is uniform, translational and potential correct to small terms of the second order, we have

$$\frac{\partial^2\Phi'}{\partial y^2} + \frac{\partial^2\Phi'}{\partial z^2} - (M_\infty^2 - 1)\frac{\partial^2\Phi'}{\partial x^2} = \frac{k-1}{Aa_\infty^2}(\dot{q} + i_{\text{от}}\dot{m}) + \dot{m}.$$

It follows from this equation that the influence of a steady supply of mass can be simulated by supplying heat to the main flow.

The analogy between heat sources and mass sources allows one to use the known methods developed for flows with heat sources to solve problems concerning supersonic flow with supply of mass and heat.

Perturbations of pressure and density in the flow due to a small addition of mass and heat may be represented correct to small terms of the second order as follows:

$$\frac{p'}{p_\infty} = -kM_\infty^2\frac{w_x'}{w_\infty} + \frac{k(k-1)}{2}M_\infty^2\frac{w_x'}{w_\infty}\frac{\Delta\dot{q} + i_{\text{от}}\Delta\dot{m}}{Aa_\infty^2} + \frac{k}{2}\left(M_\infty^2\frac{w_x'}{w_\infty}\right)^2 + \ldots$$

$$\frac{\rho'}{\rho_\infty} = -(k-1)\frac{\Delta\dot{q} + i_{\text{от}}\Delta\dot{m}}{Aa_\infty^2} - M_\infty^2\frac{w_x'}{w_\infty} -$$

$$-k\left(\frac{\Delta\dot{q} + i_{\text{от}}\Delta\dot{m}}{Aa_\infty^2}\right)^2 + kM_\infty^2\frac{w_x'}{w_\infty}\frac{\Delta\dot{q} + i_{\text{от}}\Delta\dot{m}}{Aa_\infty^2} + \ldots$$

The flow velocity perturbations are determined by solving the potential equation for the velocity perturbations with given initial and boundary conditions and a known law of mass and heat transfer.

In the case of a point source of mass and heat we have

$$w_x' = -\frac{1}{2\sqrt{M_\infty^2 - 1}}\left(\frac{k-1}{Aa_\infty^2}C_1 + C_2\right)\times$$

$$\times\left[\delta\left(x + y\sqrt{M_\infty^2 - 1}\right) + \delta\left(x - y\sqrt{M_\infty^2 - 1}\right)\right],$$

$$w_y' = -\frac{1}{2}\left(\frac{k-1}{Aa_\infty^2}C_1 + C_2\right)\left[\delta\left(x + y\sqrt{M_\infty^2 - 1}\right) - \delta\left(x - y\sqrt{M_\infty^2 - 1}\right)\right],$$

$$C_1 = \text{const}, \quad C_2 = \text{const}.$$

In the case of mass and heat addition in a supersonic flow past a curved surface given by the equation $y = h(x)$, we obtain the following relations for the velocity perturbations at the wall:

$$w'_x = -\frac{w_\infty}{\sqrt{M_\infty^2 - 1}}\,\frac{d}{dx}\,h(x) - \frac{k-1}{Aa_\infty^2}\,\frac{1}{(M_\infty^2 - 1)} \times$$

$$\times \int_{-\infty}^{x} \dot{q}\left[\mu,\ h(x) + \frac{x-\mu}{\sqrt{M_\infty^2 - 1}}\right] d\mu - \frac{1}{(M_\infty^2 - 1)} \times \int_{-\infty}^{x} \dot{m}\left[\mu,\ h(x) + \frac{x-\mu}{\sqrt{M_\infty^2 - 1}}\right] d\mu,$$

$$w'_y = w_\infty \frac{d}{dx}\,h(x) + \frac{k-1}{Aa_\infty^2}\,\frac{1}{\sqrt{M_\infty^2 - 1}} \times$$

$$\times \int_{-\infty}^{x} \dot{q}\left[\mu,\ h(x) - \frac{x-\mu}{\sqrt{M_\infty^2 - 1}}\right] d\mu + \frac{1}{\sqrt{M_\infty^2 - 1}} \times$$

$$\times \int_{-\infty}^{x} \dot{m}\left[\mu,\ h(x) - \frac{x-\mu}{\sqrt{M_\infty^2 - 1}}\right] d\mu.$$

Expressions for the velocity perturbations in flow over a surface may be obtained by the analogous method given in [1].

NOTATION

p – pressure; ρ – density; T – temperature; R – gas constant; k – adiabatic exponent; a – speed of sound; c_p – specific heat at constant pressure; w_x, w_y, w_z – velocity components along the rectangular axes x, y, z; $\dot{q}$, $\dot{m}$ – amount of heat and mass, respectively, supplied to the flow in unit time referred to unit mass of gas; Δq and Δm – amount of heat and mass, respectively, supplied to unit mass of gas of the volume in question; i_{0T} – enthalpy of added mass; A – mechanical equivalent of heat; t – time; Φ – velocity potential. Subscripts: ∞ – free-stream parameters; prime – perturbations of corresponding parameter.

REFERENCE

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20 August 1964

The Dnepropetrovsk 300-Year Reunification of the Ukraine with Russia State University

UDC 536.41

TEMPERATURE STRESSES IN A LONG PRISM OF RECTANGULAR CROSS SECTION

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Inzhenerno-Fizicheskii Zhurnal, Vol. 8, No. 5, pp. 687-691, 1965

Following publication of my article [1] on determination of the temperature stresses in a long prism of rectangular cross section, R. S. Minasyan (Erevan) pointed out that conditions (7)-(10)* are not satisfied on the interval (0, 1), except possibly for a finite number of points.

To obtain the conditions which the functions $\varphi_k(\pm 1)$ and $\psi_k(\pm 1)$, must satisfy, it is necessary to integrate (from 0 to 1) the system of equations preceding (7)-(10), namely,

*The numbering is that of [1].